7. ŋ

Dynamics
Module - 4

Science

It may be defined as the growth of ideas through observation and experimentation. Science means knowledge.

Applied Science

It is a branch of science which coordinates the research work for practical utility and services to the mankind.

Engineering Mechanics

It is the branch of applied science which deals with different laws and principles and their application to engineering problems.

Engineering Mechanics is one of the basic subject, it develops the thinking and imaginative skills of students irrespective of branches. It supports many other subjects to apply the engineering concept for manufacturing of various products and projects such as automobiles, aircrafts, electric motors robots, construction of roadways, bridges, dams, power transmission towers, projection of missiles, satellites etc.

Engineering Mechanics is divided into two - statics and dynamics.

Statics

It deals with forces and their effects on bodies at rest.

Dynamics

It deals with forces and their effects on bodies in motion.

Dynamics is again divided into two - kinetics and kinematics.

Kinetics

It is the study of bodies in motion when the forces which cause the motion are also considered.

Kinematics

It deals with study of bodies in motion when the forces which cause the motion are not considered

Motion

A body is said to be in motion if its position changes with respect to its surroundings.

A body is said to be in rest if it occupies the same position with respect to its surroundings.

Linear Motion

The motion of a body in a straight line is known as linear motion / one dimensional motion (rectilinear motion)

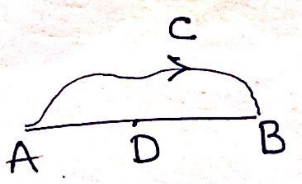
eg: A car moving along a straight road, a stone thrown vertically upwards.

Distance

The actual length of the path travelled by a body in a given interval of time is known as the distance travelled by the body. It is a scalar quantity, unit is metre

Displacement

It is the change of position of the body in a specified direction. It is a vector quantity. Unit is metre



ACB - Distance travelled

ADB - Displacement

Speed

The rate of change of distance with respect to the surroundings in a particular time is known as

Speed.

$$\text{Speed} = \frac{\text{distance}}{\text{time}}$$

it is a scalar quantity.

SI unit is m/s,

Velocity

Velocity of ~~the~~ the body may be defined as the rate of change of displacement with respect to its surroundings in a particular direction. vector - m/s

$$\text{Velocity} = \frac{\text{Displacement}}{\text{Time}} \quad \text{or} \quad V = \frac{ds}{dt} \text{ mathematically}$$

Acceleration

It can be defined as the rate of change of velocity of the body. Unit is m/s^2.

$$\text{Acceleration} = \frac{\text{Velocity}}{\text{Time.}}$$

$$a = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{ds}{dt}\right) = \frac{d^2s}{dt^2}$$

$$a = \frac{dv}{dt} \times \frac{ds}{ds} = \frac{ds}{dt} \times \frac{dv}{ds} = V\frac{dv}{ds}$$

Acceleration is said to be '+ve' when the velocity of the body increases with time and it is said to be '-ve' when the velocity of the body decreases with time. The '-ve' acceleration is also known as deceleration or retardation.

Uniform acceleration -

If the velocity of the body changes equally in equal intervals of time, the body is said to move with uniform acceleration.

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Equations of Motion in a Straight line

Let us consider a body which is moving with initial velocity 'u' in a straight line. After a small interval of time 't', its velocity changes to 'v'. Let 'S' be the displacement and 'a' be the acceleration.

Equation for Final Velocity

$$\text{Acceleration} = \frac{\text{Change of velocity}}{\text{Time taken}} = \frac{\text{Final velocity} - \text{Initial velocity}}{\text{time}}$$

ie $$a = \frac{V-u}{t}$$

$$V-u = at$$

$$V = u + at$$

Equation for Displacement

$$S = \text{average velocity} \times t$$

$$= \left(\frac{u+V}{2}\right) \times t = \left(\frac{u+u+at}{2}\right) \times t$$

$$= \left(\frac{2u+at}{2}\right) t$$

$$S = ut + \frac{1}{2}at^2$$

Derivation of $V^2 - u^2 = 2aS$

$$S = \text{average velocity} \times \text{time}$$

$$= \left(\frac{u+V}{2}\right) \times t$$

$V = u + at$

$\therefore t = \frac{V-u}{a}$

$$= \left(\frac{u+V}{2}\right) \times \left(\frac{V-u}{a}\right)$$

$$= \frac{V^2 - u^2}{2a}$$

$$\therefore V^2 - u^2 = 2aS$$

Distance travelled in the nth second.

Let Sn be the distance travelled by the body in n seconds

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and let S_{n-1} be the distance travelled by the body in the (n-1) seconds. Then distance travelled in the n^{th} sec

$$S_{n^{th}} = S_n - S_{(n-1)}$$

$$S_n = un + \frac{1}{2}an^2$$

$$S_{(n-1)} = u(n-1) + \frac{1}{2}a(n-1)^2$$

$$= \cancel{un + \frac{1}{2}an^2 - un}$$

$$S_{n^{th}} = un + \frac{1}{2}an^2 - \left[u(n-1) + \frac{1}{2}a(n-1)^2\right]$$

$$= un + \frac{1}{2}an^2 - un + u - \frac{1}{2}a(n^2 - 2n + 1)$$

$$= \frac{1}{2}an^2 + u - \frac{1}{2}an^2 + an - \frac{1}{2}a$$

$$= u + an - \frac{1}{2}a$$

$$\boxed{S_{n^{th}} = u + \frac{a}{2}(2n-1)}$$

Motion under force of gravity

When the body is freely falling under gravity the eqns. of the motion are

$$V = u + gt$$

$$S = ut + \frac{1}{2}gt^2$$

$$V^2 = u^2 + 2gS$$

When the body is moving against the force of gravity then $a = -g$

$$\therefore V = u - gt$$

$$S = ut - \frac{1}{2}gt^2$$

$$V^2 = u^2 - 2gS.$$

Note : Velocity = 0 at the maximum height. Also time taken to reach the max. ht. from the point of projection during upward motion is equal to the time taken by the body to reach the same level of projection from max. ht. during the downward motion. When the body is projected upwards the velocity of the body during the downward motion at the same level of the point of projection is equal to the velocity of projection of the body.

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Kinetics of Particles.

Mass - (m) - kg - Scalar

The quantity of matter contained in a body is known as mass of the body.

Weight (W) - N - Vector

Weight of a body is defined as the force by which the body is attracted towards the centre of the earth,

$$W = mg$$

Force (F) - N - Vector

Force may be defined as an agent which produces or tends to produce/destroys or tends to destroy motion.

$$F = ma$$

ie 1N force is that force which produces an acceleration of $1 m/s^2$ on a body of mass 1kg.

Momentum

It is the total motion possesed by a body ie. momentum of a body is given by the product of mass and velocity of the body.

$$P = mv$$ unit is kgm/s

Inertia

It is an inherent property of the body by which it offers reluctance or laziness to change its state of rest or uniform motion.

Newton's Laws of Linear motion

When a body is at rest or moving in a straight line body obeys certain laws of motion. These laws are called Newton's laws of linear motion.

First law

It states that "every body continues in its state of rest or of uniform motion in a straight line unless it is acted upon by some external force."

Second law

It states that the rate of change of momentum of a body is directly proportional to external force applied on the body and takes place in the direction of force."

Consider a body of mass 'm' moving with an initial velocity 'u' and acted upon by an external force 'F'. Let V be the final velocity after time 't'.

Initial momentum $= mu$

Final momentum $= mV$

Change in momentum $= m(V-u)$

$\therefore$ Rate of change in momentum $= \dfrac{\text{change in momentum}}{\text{time}}$

$$= \frac{m(V-u)}{t}$$

$$= ma$$

By Newton's second law

$$F \propto ma$$

$$F = kma$$

When $F = 1N$, $m = 1kg$ and $a = 1m/s^2$, then $k = 1$

$$\underline{\underline{F = ma}}$$

Third law

It states that 'to every action there is always an equal and opposite reaction'.

D' Alembert's principle of plane motion.

Let a body of mass 'm' is moving with a uniform acceleration 'a' under the action of external force F. According to the Newton's second law of motion

$$F = ma$$

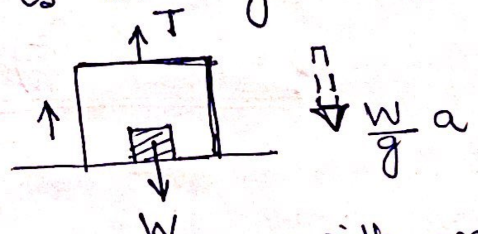
$$F - ma = 0 \quad — ①$$

From eqn ① it is clear that by applying a force '$-ma$' on the body, the body will be in equillibrium as the sum of all the forces acting is zero. Such an equillibrium is dynamic equillibrium and force 'ma' is called D' Alembert's force or reverse effective force/inertia force.

D' Alembert's principle states that "the net external forces acting on the system and the resultant reverse effective force or inertia force are in equillibrium.

Lift Motion

Case I When the lift is moving upwards.



Consider a lift moving up with an acceleration 'a'. Let 'T' be the tension in the cable supporting the lift.

W = weight of lift (W_1) + weight carried out by the lift (W_2)

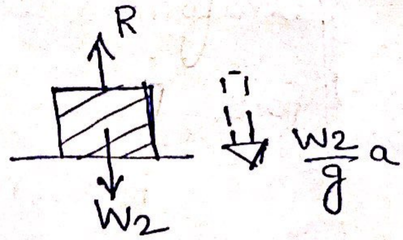
m = mass of lift (m_1) + mass carried by lift (m_2)

As the lift is moving up the net force $F = T - W$

By Newton's 2nd law, $F = ma$

$$T - W = \frac{W}{g} a \qquad \text{or} \qquad T - W - \frac{W}{g} = 0$$

$$\underline{\underline{T = W\left(1 + \frac{a}{g}\right)}}$$



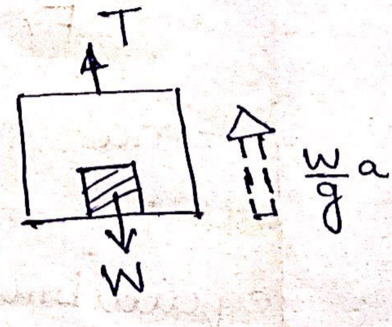
Reaction R <u>on the floor of the lift</u>

Net force $F = R - W_2$

By Newton's II Law, $F = m_2 a$

$$R - W_2 = \frac{W_2}{g} a \qquad \text{or } R - W_2 - \frac{W_2}{g} a = 0$$

$$\underline{\underline{R = W_2\left(1 + \frac{a}{g}\right)}}$$



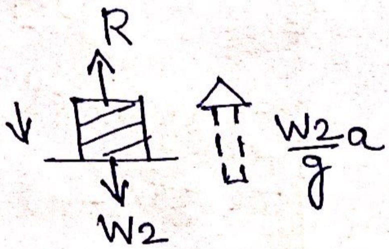
Case (ii) <u>Lift is moving downwards</u>

Net Force in the direction of motion

$$F = W - T$$

By Newton's II Law, $W - T = \frac{W}{g} a$

$$\text{or } W - T - \frac{W}{g} a = 0 \qquad \underline{\underline{T = W\left(1 - \frac{a}{g}\right)}}$$



<u>Reaction on the floor</u> of the lift

Net Force $F = W_2 - R$

By Newton's II Law, $F = ma$

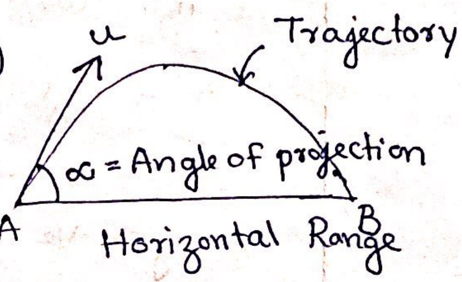
$$W_2 - R = ma$$

$$W_2 - R = \frac{W_2}{g} a \qquad \text{or } W_2 - R - \frac{W_2}{g} a = 0$$

$$\underline{\underline{R = W_2\left(1 - \frac{a}{g}\right)}}$$

Projectiles

When a particle is projected upwards at a certain angle (but not vertical), the particle traces some path in the air and falls on the ground at a point, other than the point of projection. The path traced by the particle in air is known as trajectory of the particle whereas the particle is called a projectile. The path traced by the particle is parabolic.



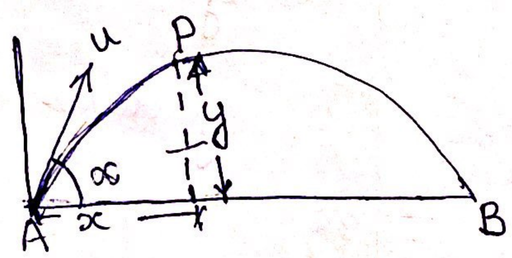
Velocity of Projection : The velocity (u) with which a projectile is projected into space.

Angle of Projection
The angle with the horizontal, at which a projectile is projected. (α)

Time of flight :- Total time taken by a projectile for which the projectile remains in space (T)

Horizontal Range :- The horizontal distance between the point of projection and the point where projectile strikes the ground is called horizontal range. (R)

Equations for the path of a projectile



Let A particle is projected upwards at an angle α with horizontal with initial velocity 'u' m/s. from point A.

The velocity of projection (u) is resolved into horizontal component and vertical component.

Horizontal component of velocity $= u\cos\alpha$

Vertical component of velocity $= u\sin\alpha$

The vertical component of the velocity is affected by gravity but the hzl. component of the velocity will remain constant (provided the air resistance is neglected.)

The particle will move in air certain path APB and will fall down at B. Let x and y = the co-ordinates of P with respect to A. and P a point on the path of the particle.

The hzl. distance travelled by the particle

x = Hzl. component of velocity $\times t$

$= (u\cos\alpha) \times t$ — ①

y = Vertical component of velocity $\times t - \frac{1}{2}gt^2$ (using $s = ut - \frac{1}{2}gt^2$)

$= (u\sin\alpha) \times t - \frac{1}{2}gt^2$ — ②

From ① $t = \frac{x}{u\cos\alpha}$

② → $y = (u\sin\alpha) \times \frac{x}{u\cos\alpha} - \frac{1}{2}g \times \left(\frac{x}{u\cos\alpha}\right)^2$

$$y = \frac{x\sin\alpha}{\cos\alpha} - \frac{gx^2}{2u^2\cos^2\alpha}$$

$$y = x\tan\alpha - \frac{gx^2}{2u^2\cos^2\alpha} \quad \text{— ③}$$

ie y varies with the square of x, hence it is the equation of a parabola. Hence path travelled is a parabola and eqn. ③ is known as equation of trajectory.

Maximum height attained by the Projectile (h_{max})

Particle reaches max. ht. when the vtl. component of velocity of projection ($u\sin\alpha$) becomes zero.

Initial velocity in vtl. direction $= u\sin\alpha$

Final velocity in vtl. direction $= 0$

Let h_{max} is the maximum height attained,

$$v^2 - u^2 = 2aS \qquad (\because a = -g \text{ and } S = h_{max})$$

$$-u^2\sin^2\alpha = -2g \times h_{max}$$

$$h_{max} = \frac{u^2\sin^2\alpha}{2g}$$

Time of flight

Time taken to reach from point A to point B.

Let T be the time of flight.

For any point on trajectory, $y = (u\sin\alpha) \times t - \frac{1}{2}gt^2$

But when the projectile reaches, B, $y = 0$ and $t = T$

$$\therefore 0 = (u\sin\alpha) \times T - \frac{1}{2}gT^2$$

$$(u\sin\alpha) \times T = \frac{1}{2}gT^2$$

$$(u\sin\alpha) = \frac{1}{2}gT$$

$$T = \frac{2u\sin\alpha}{g}$$

Horizontal Range of the Projectile (R)

This is the hzl. distance AB.

R = Velocity component in hzl. direction × Time taken to reach from A to B.

$$= (u\cos\alpha) \times T$$

$$= (u\cos\alpha) \times \frac{2u\sin\alpha}{g}$$

$$= \frac{u^2}{g} \times 2\sin\alpha\cos\alpha \qquad (\because 2\sin\alpha\cos\alpha = \sin 2\alpha)$$

$$R = \frac{u^2}{g} \times \sin 2\alpha$$

Angle of projection (α) for maximum horizontal range

Hzl. range (R) is maximum, when $\sin 2\alpha$ is maximum

$\sin(2\alpha) = 1$ or $\sin 2\alpha = \sin 90°$ $\quad \therefore \alpha = \frac{90}{2} = 45°$

∴ Maximum hzl. range (R_{max}) by substituting $\alpha = 45$

$$R_{max} = \frac{u^2}{g}\sin(2\times45) = \frac{u^2}{g}$$

Time to reach the Highest Point.

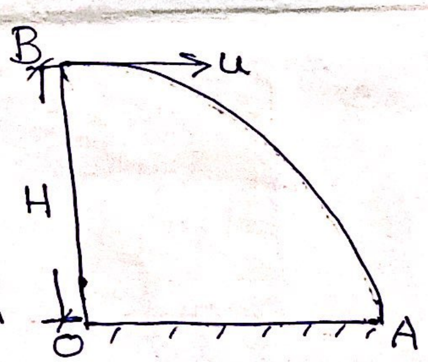
When reaches the highest point, initial velocity in the vertical direction is zero. T' time taken by projectile to reach highest point.

$V = u + at$ $\quad (\because a = -g)$

$0 = u\sin\alpha - g\times T'$

$\therefore T' = \frac{u\sin\alpha}{g}$ ie T' = half the time of flight.

Motion of a body thrown horizontally from a given height into air.



u = hzl. velocity with which body is thrown.

B = Pt. at which body is thrown.

A = Pt. at which the body strikes the ground.

At B, hzl. velocity = u & vertical velocity V=0

Hzl. component u remains constant from B to A

Vtl. component V is influenced by g.

∴ Resultant velocity $= \sqrt{u^2+v^2}$ and $\tan\theta = \frac{V}{u}$ → [resultant with hzl.]

The vtl. downward distance (H) = (vtl comp. at B) $\times t + \frac{1}{2}gt^2$

$= 0\times t + \frac{1}{2}gt^2 = \frac{1}{2}gt^2$

$H = \frac{1}{2}gt^2$ 't time to travel from A to B

The Range = (hzl. comp) $\times t$ = ut

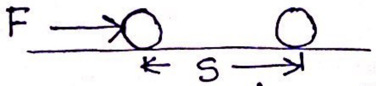
Final vtl. velocity at A = Initial vertical velocity + gt

$V = 0 + gt = gt$.

The resultant velocity at A $= \sqrt{u^2+V^2}$

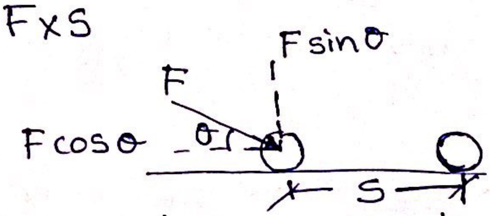
Work, Energy

Work : Whenever a force acts on a body and body undergoes a displacement, some work is said to be done. ie Work is defined as the product of force and distance moved by the body in the direction of force.



If a force F is acting on a body and body moves a distance S in the direction of force, then workdone is given by

$$W = F \times S$$



If a force acting on the body and the distance moved are not in the same direction then

$$W = F\cos\theta \times S$$

ie W = Component of force in the direction of desplacement × displacement.

When $\theta = 0$, $W = F \cdot S$

$\theta = 90^\circ$, $W = 0$

Unit of workdone = Nm

1 J or 1 Nm is the workdone by a force of 1 N when it displaces the body through 1 m.

Power : It is the rate of doing work or workdone per second.

Unit is Watts or Nm/s or J/S

$$P = \frac{W}{t} = \frac{F \times S}{t} = F \cdot V$$

Energy : Energy is defined as the capacity to do work. It is the product of power and time.

Unit is J or Nm.

Energy can be expressed in many forms like: mechanical energy, heat energy etc. In engineering mechanics, we are dealing with mechanical energy. The two types are i) kinetic ii) Potential energy. Depending upon the state of rest or that of motion, the body may posses anyone or both of the above energies.

Potential energy

Energy possessed by a body due to the position or elevation with respect to the reference or datum.

Consider a body of mass 'm' kg elevated through a distance 'h' m above the surface of earth.

$PE = mgh$

PE = workdone against gravity

When the height of mass changes from h_1 to h_2, then change in PE, $\Delta PE = mg(h_2 - h_1)$

Kinetic Energy

It is the energy possessed by the body by virtue of its velocity or motion.

KE possessed by a body moving with a velocity

$V = \frac{1}{2}mv^2$.

Work Energy Principle.

The work energy principle states that workdone by net external force is equal to the change in KE.

Consider a body of mass 'm' acted upon by an external force F. Let the initial velocity of the body u changes to V due to F within a distance 'S'. Let the time taken is 't'

Work done by external force = F.S

By Newton's Second Law $F = ma$

$$W = ma \cdot S \quad -①$$

From equation of motion, $v^2 = u^2 + 2as$

$$as = \frac{v^2 - u^2}{2}$$

Substituting as in ① $\quad W = m\frac{(v^2-u^2)}{2}$

$$W = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$$

$$W = \Delta KE$$

Law of Conservation of Linear Momentum.

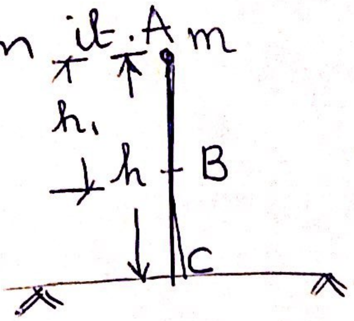
It states that if the resultant of external forces acting on a system is zero, the momentum of the system remains constant. It means that the total momentum of the system before collision is equal to the total momentum of the system after collision.

Consider a system consisting of two bodies having masses m_1 and m_2, let u_1 and u_2 be the initial velocities before collision and V_1 & V_2 be final velocities after collision. According to law of conservation of linear momentum, momentum before collision = momentum after collision.

$$m_1u_1 + m_2u_2 = m_1V_1 + m_2V_2$$

Law of conservation of energy

It states that " energy can neither be created nor be destroyed but it can only be transformed from one form to another". It also states that total energy remains constant provided no energy is added or taken away from it.



Consider a body freely falling from rest from a height 'h' above the ground.

At level A, KE = 0

$PE = mgh$

$\therefore$ Total energy at A = KE + PE = $\underline{\underline{mgh}}$

At level B, $KE = \frac{1}{2} m V_B^2$

using $v^2 = u^2 + 2gs$

$V_B^2 = 0 + 2gh_1 = 2gh_1$

$KE = \frac{1}{2} m \times 2gh_1 = mgh_1$

$PE = mg(h - h_1)$

Total energy at B = KE + PE

$= mg(h + h_1 - h_1) = \underline{\underline{mgh}}$

At level C, $KE = \frac{1}{2} m V_C^2$ $\quad V_C^2 = 0 + 2gh$

$= \frac{1}{2} m \times 2gh = mgh$

$PE = 0$

Total energy = $\underline{\underline{mgh}}$

Impulsive Force

It is a very large force acting on a body for a very short period of time.

Impulse :- The whole effect of impulsive force is measured by impulse. It is the product of force and the time during which the force act on the body.

$I = F \times t$

unit is Ns.

Impulse Momentum principle.

It states that the component of resultant linear impulse along any direction is equal to the change in the component of momentum in that direction

Let a force F is acting on a body of mass 'm' and 't' time for which the force acts on the body.

Impulse of force on the body $I = F \times t$

According to Newton's II^{nd} Law

$$F = ma$$

$$= \frac{m(v-u)}{t}$$

$$I = F \times t = mv - mu = \text{change in momentum.}$$

Collision of elastic bodies.

Whenever two elastic bodies collide with each other they tend to compress each other, immediately after this the two bodies attempt to regain its original shape due to their elasticity. This process of regaining the original shape is called restitution.

Time of compression

It is the time taken by the two bodies in compression after the instant of collision.

Time of restitution

It is the time taken by two bodies to regain the original shape after compression.

Time of collision

The sum of time of compression and time of restitution is known as period of collision or period of impact.

Types of impact

1) Direct impact
2) Indirect impact (Oblique impact)

Impact means the collision of two bodies which occur in a very small interval of time and during which the two bodies exert very large force on each other.

The important types of impacts are direct and indirect impact.

Direct impact of two bodies

The collision between two bodies is known as direct impact, if the two bodies before impact are moving along the line of impact.

The two bodies A and B are moving in a horizontal line before collision in the same direction,

Let m_1 - mass of body A
m_2 - mass of body B
u_1 - initial velocity of body A
u_2 - initial velocity of body B
V_1 - Velocity of A after collision.
V_2 - velocity of B after collision.

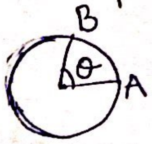
Then $m_1u_1 + m_2u_2 = m_1V_1 + m_2V_2$

Module - 5

Rotational Motion.

Angular velocity (ω) - (rad/s)

The rate of change of angular displacement is known as angular velocity.



$$\omega = \frac{d\theta}{dt}$$

Angular velocity can also be expressed in terms of revolutions per minute. ie N revolutions per minute is

$$2\pi N \text{ rad/m} = \frac{2\pi N}{60} \text{ rad/s.}$$

Angular acceleration (α) - (rad/s^2)

It is the rate of change of angular velocity.

It can be +ve, -ve or uniform and variable.

$$\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$$

$$= \frac{d\omega}{dt} \times \frac{d\theta}{d\theta} = \frac{d\omega}{d\theta} \cdot \frac{d\theta}{dt} = \omega \frac{d\omega}{d\theta}$$

Motion of rotation under uniform circular acceleration.

$$\omega = \omega_0 + \alpha t$$

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$$

$$\omega^2 = \omega_0^2 + 2\alpha\theta$$

$$\theta_{nth} = \omega_0 + \frac{1}{2}\alpha(2n-1)$$

Kinetics of Rotation.

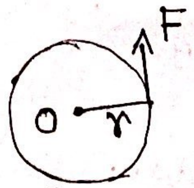
The motion of a rigid body in which the particles move in a circular path is called rotational motion or circular motion.

Torque (τ)

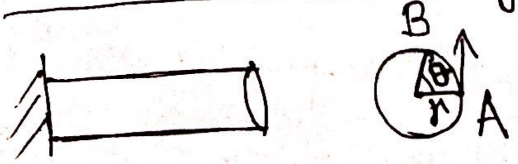
It is the turning or twisting moment of force on the body in which it acts.

τ = product of force and $\perp^r$ distance between the line of action of force and central point.

$$\tau = F \times r$$



Power transmitted by a shaft.



Consider a shaft rotating due to a tangential force F applied at a distance 'r' from centre 'O'. Let the body rotate through a small angle θ as the result of the force and with angular velocity 'ω'. Workdone by the torque

$$= F \times AB$$
$$= F \times r\theta$$
$$= \tau\theta$$

$\therefore$ Power transmitted by shaft $= \dfrac{\text{Work done}}{\text{time}} = \dfrac{\tau\theta}{t}$

$$= \tau\omega \text{ watts.}$$

Angular momentum

It is the total motion possessed by a rotating body. It is also called moment of momentum. It is equal to the product of mass moment of inertia and angular velocity.

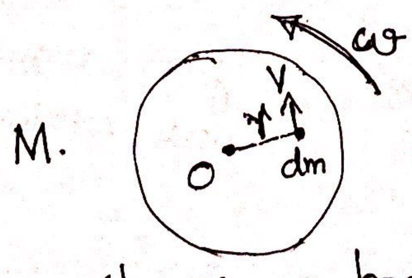


Figure shows a body of mass 'm' rotating with an angular velocity 'ω'. Consider an elementary mass 'dm' at a distance 'r' from 'O'

Momentum of the element 'dm' about O $= dm \times v$

$= dm \times r\omega$ $\qquad v = r\omega$

Moment of momentum of the element $= dm\, r\omega \times r$

$= dm r^2 \omega$

$\therefore$ Moment of momentum or angular momentum of the entire body $= \int dm r^2 \omega$

$= \omega \int dm r^2$

$= \omega I = \underline{\underline{I\omega}}$

Newton's Laws of rotational motion.

1st Law :- Every body continues in its state of rest or of uniform motion of rotation about an axis unless it is acted upon by some external torque.

2nd Law :- It states that the rate of change of angular momentum of a body is directly proportional to the external torque and takes place in the direction in which the torque acts.

Consider a body acted upon by a torque τ, Let the angular velocity is changing from ω_0 to ω in a time of 't' seconds, let I be the mass moment of inertia and α the angular acceleration.

Change in angular momentum $= I\omega - I\omega_0$

Rate of change of angular momentum $= \dfrac{I(\omega - \omega_0)}{t} = I\alpha$

By Newton's law of rotation, $T = I\alpha$

3rd Law :- To every torque there is always an equal and opposite torque.